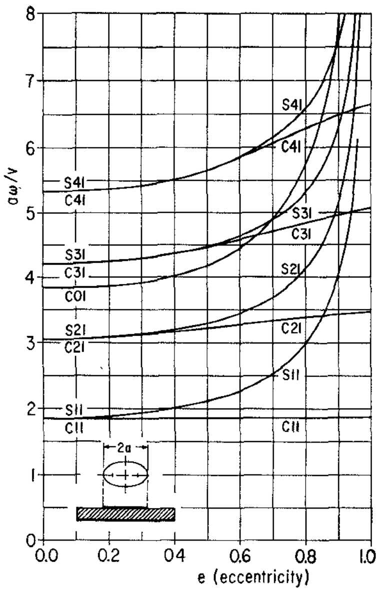

Fig. 2. Mode chart for the elliptic microstrip resonator.

with

- e eccentricity of the cross section,
- ω radial frequency,
- v $=(\mu\epsilon)^{-1/2}$,
- k wavenumber.

It is interesting to note that these eigenvalues are equal to the eigenvalues of TE modes in a hollow conducting elliptical waveguide with the same major axis $2a$ and eccentricity e. This means that the approximate formulas for the TE_{c01}, TE_{c11}, TE_{s11}, TE_{c21}, and TE_{s21} modes given in [4] can be used for the corresponding TM modes in the resonator described here.

It is not true that a numerical solution of (1a) and (1b) has only been possible for the first 1st-order even and odd mode, which was proven in recently published work [3]–[5], [7], [8].

For a comparative study of the different possibilities to compute modified Mathieu functions we refer to McLachlan's standard work on Mathieu functions [2].

Mode Chart

Using a Bessel-function product series, (1a) and (1b) were solved for the successive even and odd modes [3], [4]. Formula (2a) can be written as

$$a\omega/v = 2\sqrt{q}/e \tag{2b}$$

and, as q only depends upon the mode and eccentricity [4], it is possible to construct a general mode chart as given in Fig. 2. This theoretical result confirms the measured values reported in [1], although the 31 and 41 modes were not observed in the experiments reported in the latter. The fact that the measured resonant frequency is smaller than the theoretical one is due to the fringe field at the edge of the resonator, and the actual differences are a function of the mode, the dielectric medium, and the dimensions.

Unloaded Quality Factor

Taking into account that the magnetic field does not depend upon the z coordinate, and following the same procedure as described in [6], it is easily shown that the unloaded Q is given by

$$Q_0 = H(\pi f\sigma/\mu)^{1/2} \tag{3}$$

with

- H substrate thickness,
- σ conductivity of the metal strip,
- f resonant frequency.

As the resonant frequency depends upon the mode, the substrate, the major axis, and the eccentricity (2b) it is obvious that the quality factor is a function of those parameters too. This means that in comparison with the circular case described in [6] we have one additional degree of freedom, namely the eccentricity value. This can be quite important for practical applications, as was pointed out by Irish [1].

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Efficiencies of Microwave 2-Ports from Reflection Coefficient Measurements

R. W. BEATTY

***Abstract*—It is well-known that one can determine the efficiency of a microwave 2-port by measuring the reflection coefficient Γ_1 at the input port when the output port is terminated by a sliding short circuit. The locus of Γ_1 is a circle whose radius equals the efficiency η_{2m} for energy entering port 2 when port 1 is terminated in a nonreflecting load. Similarly η_{1m} is the radius of the Γ_2 circle when port 1 is terminated in a sliding short circuit.**

This note describes a procedure for obtaining, from the same measured data, new reflection coefficients Γ_{1N} and Γ_{2N}, whose circular loci have radii R_{1N} and R_{2N} which give the efficiencies of the 2-port when connected to an arbitrary load of reflection coefficient Γ_L.

Thus the Γ_1 or Γ_2 data may be used to obtain the efficiency of the 2-port when terminated in any arbitrary load. The method is potentially more accurate than the 3-point method since errors can be reduced by drawing a circle through many measured points.

Introduction

It has been shown [1] that the efficiency of a 2-arm waveguide junction or 2-port terminated in a nonreflecting load is given by

$$\eta_{1m} = \frac{Z_{01}}{Z_{02}} \frac{|S_{21}|^2}{1-|S_{11}|^2}$$

and

$$\eta_{2m} = \frac{Z_{02}}{Z_{01}} \frac{|S_{12}|^2}{1-|S_{22}|^2} \tag{1}$$

where η_1 corresponds to feeding arm 1 and terminating arm 2, while η_2 corresponds to feeding arm 2 and terminating arm 1. Providing that the waveguide junction is linear, passive, and obeys the reciprocity condition $Z_{01}S_{21}=Z_{02}S_{12}$, the above efficiencies may be obtained [2], [3] from measurements of the reflection coefficient at one port when the other port is terminated by a sliding short circuit in a section of lossless waveguide. The locus of the reflection coefficients so obtained are circles. It has been shown [1] that η_{2m} equals the radius R_1 of the Γ circle and that η_{1m} equals the radius R_2 of the Γ_2 circle. This is illustrated in the diagrams of Fig. 1.

It is perhaps not so well-known that from the same data, one can obtain the efficiencies of the 2-port for any other termination besides the nonreflecting termination [4]. One way in which this can be done is by transforming the measured reflection coefficients to new values

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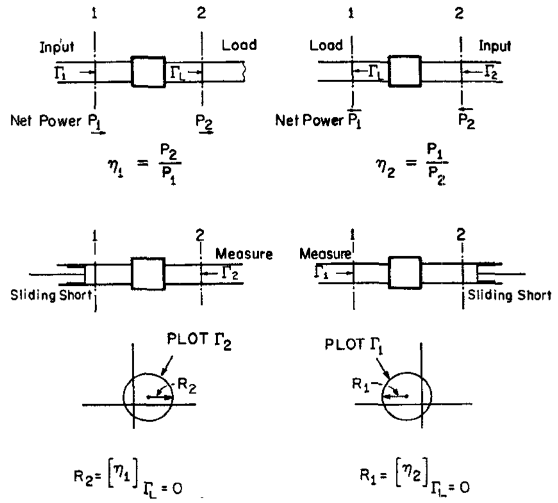

Fig. 1. Diagrams illustrating graphical method of obtaining 2-port efficiencies from reflection coefficient measurements.

and measuring the radii of their circular loci. These radii can be shown to be closely related to the desired efficiencies.

Procedure

Consider that Γ_1 has been measured corresponding to various positions of the sliding short circuit in arm 2 of a 2-arm waveguide junction. A circle has been drawn through the measured points. The radius of the Γ_1 circle equals η_{2m}. Now suppose that we wish to determine η_2 for a different termination on arm 1 having a reflection coefficient Γ_L. We use the data for Γ_1 and calculate new reflection coefficients Γ_{1N} by the formula

$$\Gamma_{1N} = \frac{\Gamma_1 - \Gamma_L}{1 - \Gamma_1\Gamma_L}. \qquad (2)$$

Since Γ_{1N} is a linear fractional transformation of Γ_1, it has a circular locus of radius R_{1N}. One plots Γ_{1N}, measures R_{1N}, then calculates

$$\eta_2 = \frac{R_{1N}}{\sqrt{1 + \left(\frac{2|\Gamma_L \sin \psi_L|}{1 - |\Gamma_L|^2}\right)^2}} \qquad (3)$$

where ψ_L is the phase of Γ_L.

In a similar way one may obtain from data on Γ_2,

$$\eta_1 = \frac{R_{2N}}{\sqrt{1 + \left(\frac{2|\Gamma_L \sin \psi_L|}{1 - |\Gamma_L|^2}\right)^2}}. \qquad (4)$$

Theory

First we derive an expression for the radius of the circular locus of R_{1N}. We note that

$$\Gamma_1 = \frac{(S_{12}S_{21} - S_{11}S_{22})e^{i\phi} + S_{11}}{1 - S_{22}e^{i\phi}} \qquad (5)$$

where ϕ is the phase of the sliding short-circuit termination. Consider the following equation

$$\Gamma_{1N} = \frac{(S_{12}S_{21} - S_{11}S_{22})e^{i\phi} + S_{11} - \Gamma_L(1 - S_{22}e^{i\phi})}{1 - S_{22}{}^{i\phi} - [(S_{12}S_{21} - S_{11}S_{22})e^{i\phi} + S_{11}]\Gamma_L}. \qquad (6)$$

This can be cast in the form

$$\Gamma_{1N} = \frac{ae^{i\phi} + b}{ce^{i\phi} + d} \qquad (7)$$

and the radius of the Γ_{1N} circle is [1]

$$R_{1N} = \frac{|ad - bc|}{|d|^2 - |c|^2} = \frac{|S_{12}S_{21}(1 - \Gamma_L{}^2)|}{|1 - S_{11}\Gamma_L|^2 - |(S_{12}S_{21} - S_{11}S_{22})\Gamma_L + S_{22}|^2}. \qquad (8)$$

If we apply the reciprocity condition $Z_{01}S_{21} = Z_{02}S_{12}$

$$R_{1N} = \frac{Z_{02}}{Z_{01}} \frac{|S_{12}|^2 |1 - \Gamma_L{}^2|}{|1 - S_{11}\Gamma_L|^2 - |(S_{12}S_{21} - S_{11}S_{22})\Gamma_L + S_{22}|^2}. \qquad (9)$$

However, the efficiency η_2 has been shown [1] to be equal to the right side of (9) multiplied by $1 - |\Gamma_L|^2/|1 - \Gamma_L{}^2|$.

Thus

$$\eta_2 = R_{1N} \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_L{}^2|} = \frac{R_{1N}}{\sqrt{1 + \left(\frac{2|\Gamma_L \sin \psi_L|}{1 - |\Gamma_L|^2}\right)^2}}. \qquad (3)$$

It can be shown in a similar way that if

$$\Gamma_{2N} = \frac{\Gamma_2 - \Gamma_L}{1 - \Gamma_2\Gamma_L} \qquad (10)$$

and R_{2N} is the radius of the Γ_{2N} circle, then

$$\eta_1 = \frac{R_{2N}}{\sqrt{1 + \left(\frac{2|\Gamma_L \sin \psi_L|}{1 - |\Gamma_L|^2}\right)^2}}. \qquad (4)$$

Conclusion

We can obtain the efficiency for any termination from the data used to plot the Γ_1 circle.

The additional labor of transforming the measured reflection coefficient to a new reflection coefficient according to (2) can be easily and quickly done on a desk-top programmable electronic calculator or a time-share computer terminal. Thus the method is a simple extension of previous methods involving a small amount of additional calculation but no additional taking of data. Compared to the method described by Mathis [4], it requires slightly less data and is potentially more accurate.[1]

It is of course possible to obtain η_1 and η_2 from only three measured reflection coefficients (either Γ_1 or Γ_2) if the corresponding terminations are known. However, the accuracy of the above method using circular loci can be much better because such curve fitting tends to "average out" errors in the individual measurements.

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[1] It has been shown that from the same data, plus one additional measured point with the 2-port terminated in the arbitrarily selected load, one can obtain the efficiency corresponding to that termination. This is done by means of a graphical construction and additional calculation. See [4].

Focusing of 52-GHz Beams by Cylindrical Mirrors

J. A. ARNAUD, D. C. HOGG, AND J. T. RUSCIO

The possibility of using inexpensive commercial glass mirrors for refocusing and redirecting millimeter-wave beams [1] has been investigated; such beams would be useful for distributing large quantities of information in cities [2], [3]. Interference is expected to be minimal as a result of the close confinement of the beams. We report preliminary experiments made in the 50–55-GHz band, using a swept backward-wave oscillator (BWO) as a source.

A closed triangular path (25 m+35 m+25 m) incorporating a beam launcher, two refocuser/redirectors, and a beam collector has been set up on Crawford Hill, Holmdel, N. J. A refocuser is shown in Fig. 1.

The beam launcher and the beam collector are of the periscopic

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